Problem 1 (30%)**.** There are k people in a lift at the ground floor. Each wants to get off at a random floor of one of the n upper floors. What is the expected number of lift stops? Justify your answer.

Hint: Consider the indicator variable X_i for the events that at least one person is off at the i-th floor, and apply the linearity of expectation. The answer is $n(1-(1-1/n)^k)$.

Problem 2 (30%)**.** Let $\mathcal{F}$ be a k-uniform k-regular family, i.e., each set has k points and each point belongs to k sets. Let $k \geq 10$. Show that there exists at least one valid 2-coloring of the points.

Hint: Define proper events for the sets and apply the symmetric version of the local lemma.

Problem 3 (20%)**.** Prove that the statement of the asymmetric version of the local lemma implies the conclusion of the symmetric version. That is, assuming that $\Pr[A_i] \leq p$ for all i and $ep(d+1) \leq 1$, use the asymmetric version of the local lemma to prove that $\Pr\left[\bigcap_i \overline{A_i}\right] > 0$.

Hint: Let $x(A_i) = \frac{1}{d+1}$ for all $1 \leq i \leq n$. Use the inequality $\frac{1}{e} \leq \left(1 - \frac{1}{d+1}\right)^d$ obtained by the limit formula of $1/e$ and the fact that it converges from the above.

Problem 4 (20%)**.** Given a graph $G = (V, E)$ and a 0-1 vector $v = (v_1, v_2, \ldots, v_n) \in \{0,1\}^n$, where $n = |V|$, define the And-Or formula

$$F_G \;=\; \bigwedge_{\{i,j\} \notin E} \left(\; \overline{v_i} \;\vee\; \overline{v_j} \;\right).$$

Consider the vertex set $S_v := \{i\colon v_i = 1\}$. Show that S_v is a clique of G if and only if v satisfies the formula F_G.