# 1 The Hungarian Algorithm in $O(n^4)$ Time

In the following, we summarize the Pseudo-Code for the $O(n^4)$ time implementation of the Hungarian algorithm, using the procedures developed for the Maximum Bipartite Matching problem.

Let $G = (V, E$ be the input complete bipartite graph with partite sets $A$ and $B$, where $|A| = |B| = n$, and edge edge $w_{u,v}$ for all $u, v \in E$. The algorithm goes as follows.

1. $M \longleftarrow \emptyset$.

   For each $v \in V$, $y_v := \begin{cases} \max_{b \in B} w_{v,b}, & \text{if } v \in A, \\ 0, & \text{otherwise.} \end{cases}$.

2. For each *unmatched* vertex $u \in A$, do

   (a) Mark all the vertices as *unvisited*.

   (b) Repeat the following, until Aug-Path($u$) on $G_y = (V, E_y)$ returns true.

   - Let $S$ be the set of vertices in $A$ that are marked as *visited*.
     Let $T$ be the set of vertices in $B$ that are matched to vertices in $S \setminus \{u\}$.
   - $\epsilon \longleftarrow \min_{a \in S,\, b \in B \setminus T} (y_a + y_b - w_{a,b})$.
   - Set $y_a \longleftarrow y_a - \epsilon$ for each $a \in S$.
     Set $y_b \longleftarrow y_b + \epsilon$ for each $b \in T$.
   - Mark all vertices as *unvisited*.

3. Output $M$ as the maximum weight matching and $y$ as the minimum weight vertex cover.

Note that, we don't need to construct the graph $G_y = (V, E_y)$ explicitly. It suffices to modify the procedure Aug-Path(), so that it only follows tight edges when exploring.